

COLUMNS AND STRUTS

DEFINITION: Column is defined as member of structure which is subjected to axial compressive load.

e.g. Pillars

Type of situation based:-

Steel - post, stranchion

bridge - pier

R.C.C. - column

towers - strut

wood - post.

Strut:

If the member of the structure is not vertical and also horizontal, inclined direction of the structure is known as strut.

e.g. connecting rods, piston rods, transmission tower rods, chimneys.

Classification of columns:

1) Long column :-

long column is column which fails primarily due to bending stress

a) In long columns, direct stresses are very small compared to their buckling stresses.

b) Long column is a column whose slenderness ratio is greater than 120 (or) whose length is more than 30 times the least lateral dimension.

c) for mild steel column slenderness ratio is greater than 80.

2) short column

short column is a column which fails primarily due to direct stress.

a) in short columns, the buckling stresses are very small compared to the direct stresses.

b) in short column is a column whose slenderness ratio is less than 32 (or) whose length is less than 8 times the least lateral dimension.

c) for mild steel column slenderness ratio is less than 80.

3) medium (or) intermediate column

medium column is a column which fails either due to direct stress (or) buckling stress.

a) for medium columns slenderness ratio between 32-120.

b) for medium columns their length is more than 8 times but less than 30 times their least lateral dimension.

Effective length:-

It depends upon the end conditions and choose the effective length.

Radius of Gyration:-

The ratio square root of moment of inertia to the cross-area is called as Radius of gyration.

$$R = \sqrt{\frac{I}{A}}$$

$$(or) k = \sqrt{\frac{I}{A}}$$

Slenderness ratio:-

The ratio of effective length to least radius of gyration is called slenderness ratio. It is denoted by 'i'.

$$i = \frac{kL}{r_{min}}$$

Safe load (or) working load:-

A column (or) strut can never be subjected to critical load and the column is subjected to less than critical load.

$$\text{safe load} = \frac{\text{critical load}}{\text{factor of safety}}$$

$$f.o.s = 1.5$$

$$\text{factor of safety} = \frac{\text{critical load}}{\text{safe load}}$$

Fuler's formula for column:-

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

EI = flexural rigidity.

L = effective length.

Fuler's crippling load for different end conditions:-

1) Both ends are hinged $\rightarrow L = l$

2) Both ends are fixed $\rightarrow L = l/2$

3) One end fixed & other end is free

4) One end fixed & other end is hinged $\rightarrow L = l/\sqrt{2}$

Expression for crippling load when both ends of column are hinged:-

The load at which the column just buckles is called crippling load.

Consider a column AB of length 'l' and uniform area, hinged at both of its end A and B.

Let P be the crippling at which the column just buckles due to the crippling load, the column will be deflect into a curved form ACB as shown in fig. Consider any section at distance 'x' from A.

Let y = deflection at the section.

the moment due to crippling load at the section = $-P \cdot y$ -① [-sign in upward load]
to consider for the double integration

$$M = EI \frac{d^2y}{dx^2} -②$$

to equate the ① and ②

$$EI \frac{d^2y}{dx^2} = -P \cdot y$$

$$EI \frac{d^2y}{dx^2} + Py = 0$$

$$EI \frac{d^2y}{dx^2} + Py = 0$$

Divide the EI in b.c.

$$\frac{EI}{EI} \frac{d^2y}{dx^2} + \frac{P}{EI} y = 0$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = 0 \quad [\alpha = \sqrt{\frac{P}{EI}}, \alpha^2 = \frac{P}{EI}]$$

The solution equation is

$$y = c_1 \cos(\alpha \cdot x) + c_2 \sin(\alpha \cdot x)$$

$$y = c_1 \cos\left(\sqrt{\frac{P}{EI}} x\right) + c_2 \sin\left(\sqrt{\frac{P}{EI}} x\right)$$

$$y = c_1 \cos\left(x \sqrt{\frac{P}{EI}}\right) + c_2 \sin\left(x \sqrt{\frac{P}{EI}}\right) -③$$

i) At $x=0, y=0$ sub the eqn ③

$$0 = c_1 \cos(0) + c_2 \sin(0)$$

$$0 = c_1 \cdot 1 + 0$$

$$\boxed{c_1 = 0}$$

ii) $x=1, y=0$

sub in eqn ③

$$0 = c_1 \cos\left(1 \times \sqrt{\frac{P}{EI}}\right) + c_2 \sin\left(1 \times \sqrt{\frac{P}{EI}}\right)$$

$$= 0 + c_2 \sin\left(1 \times \sqrt{\frac{P}{EI}}\right)$$

$$c_2 = 0 -④$$

$$0 = C_2 \sin \left(2 \times \sqrt{\frac{P}{EI}} \right)$$

$$\sin \left(2 \times \sqrt{\frac{P}{EI}} \right) = 0$$

$$2 \sqrt{\frac{P}{EI}} = \sin 0^\circ \text{ (or)} \sin \pi \text{ (or)} \sin 2\pi$$

$$2 \sqrt{\frac{P}{EI}} = 0 \text{ (or)} \pi \text{ (or)} 2\pi$$

Taking the least value of

$$2 \sqrt{\frac{P}{EI}} = \pi$$

Squaring on B.S.

$$\therefore P = \pi^2 EI / l^2$$

Expression for crippling load when one end of the column fixed and other end is free

Consider a column AB of length (l) and uniform cross area,

fixed at the end A

and free at B.

Consider any section at a distance 'y' from the fixed end A.

Let y = deflection at the section.

a = deflection at the free end 'B'

The moment at the section due to

$$\text{Crippling load} = P(a-y) - ①$$

By using double integration Eqn

$$M = EI \frac{dy}{dx^2} - ②$$

Equate the equations ① and ②

$$EI \frac{dy}{dx^2} = P(a-y).$$

$$\text{Eq } \frac{d^2y}{dx^2} = Pa - Py$$

$$EI \frac{d^2y}{dx^2} + Py = Pa$$

Divide EI on both sides.

$$\frac{EI}{EI} \frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{Pa}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{Pa}{EI}$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \alpha^2 \times a, \quad \text{where } \alpha = \sqrt{\frac{P}{EI}},$$

$$\alpha^2 = \frac{P}{EI}$$

$$y = C_1 \cos \alpha x + C_2 \sin \alpha x + \alpha^2 \times a$$

$$y = C_1 \cos \left(\sqrt{\frac{P}{EI}} x \right) + C_2 \sin \left(\sqrt{\frac{P}{EI}} x \right) + a \quad \text{--- (3)}$$

To sub $x=0, y=0$ in eqn (3).

$$0 = C_1 \cos(0) + C_2 \sin(0)$$

$$0 = C_1 + a$$

$$\boxed{C_1 = -a}$$

To differentiating the eqn (3) with respect to x .

$$\frac{dy}{dx} = C_1 (-1) \sin \left[x \sqrt{\frac{P}{EI}} \right] \sqrt{\frac{P}{EI}} + C_2 \cos \left[x \sqrt{\frac{P}{EI}} \right]$$

$$\sqrt{\frac{P}{EI}} + 0.$$

$$= C_1 \sqrt{\frac{P}{EI}} \sin \left[x \sqrt{\frac{P}{EI}} \right] + C_2 \sqrt{\frac{P}{EI}} \cos \left[x \sqrt{\frac{P}{EI}} \right] \quad \text{--- (4)}$$

$$\text{Let } x=0, \frac{dy}{dx} = 0$$

$$0 = -C_1 \sqrt{\frac{P}{EI}} \sin \left[0 \times \sqrt{\frac{P}{EI}} \right] + C_2 \sqrt{\frac{P}{EI}} \cos \left[0 \times \sqrt{\frac{P}{EI}} \right]$$

$$0 = 0 + C_2 \sqrt{\frac{P}{EI}}$$

$$C_2 \sqrt{\frac{P}{EI}} = 0$$

$$\sqrt{\frac{P}{EI}} = 0$$

To sub $c_1 = -a$, $c_2 = 0$ in eqn ③

$$y = -a \cos \left[x \cdot \sqrt{\frac{P}{EI}} \right] + a \quad \text{--- ⑤}$$

But at the free end of the column

$x = l$, $y = a$ to sub the eqn ⑤

$$a = -a \cos \left(l \sqrt{\frac{P}{EI}} \right) + a$$

$$0 = -a \cos \left[l \sqrt{\frac{P}{EI}} \right]$$

$$a \cos \left[l \sqrt{\frac{P}{EI}} \right] = 0$$

But a cannot be equal to 0

$$\cos \left[l \cdot \sqrt{\frac{P}{EI}} \right] = 0 = \cos \frac{\pi}{2} \text{ (cor)} \cos \frac{3\pi}{2}, \cos \frac{5\pi}{2}$$

$$l \cdot \sqrt{\frac{P}{EI}} = \frac{\pi}{2} \text{ (cor)} \frac{3\pi}{2}$$

taking the least value

$$l \cdot \sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{2l}$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{2l}$$

$$\frac{P}{EI} = \left(\frac{\pi^2}{4l^2} \right)$$

$$P = \frac{\pi^2 EI}{4l^2}$$

Expression for crippling load when both the ends of the column are fixed:-

Consider a column AB of length 'l' and uniform cross area are fixed at both ends AB as shown in fig.

Let P is the crippling load M_0 is the fixed end moments at the ends A and B.

The moment at the section

$$= M_0 - P \cdot y \quad \text{--- (1)}$$

By using the double integration.

$$M = EI \frac{d^2y}{dx^2} \quad \text{--- (2)}$$

To equate the eqn (1) & (2)

$$EI \frac{d^2y}{dx^2} = M_0 - P \cdot y$$

$$EI \frac{d^2y}{dx^2} + P \cdot y = M_0$$

By dividing on b.s EI

$$\frac{d^2y}{EI} \frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{M_0}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{M_0}{EI} \times \frac{P}{P}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{M_0}{P} \frac{P}{EI}$$

Above the eqn. can be written as

$$\frac{d^2y}{dx^2} + \alpha^2 \cdot y = \frac{M_0}{EI \alpha^2} \quad \left[\alpha^2 = \frac{P}{EI} \text{ & } \alpha = \sqrt{\frac{P}{EI}} \right]$$

$$y = C_1 \cos(\alpha \cdot x) + C_2 \sin(\alpha x) + \frac{M_0}{EI \alpha^2}$$

$$= C_1 \cos\left(\sqrt{\frac{P}{EI}} x\right) + C_2 \sin\left(\sqrt{\frac{P}{EI}} x\right) + \frac{M_0}{EI \cdot \frac{P}{EI}}$$

$$dy = C_1 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \cdot 3 \quad (3)$$

Sub $x = 0, y = 0$

$$0 = C_1 \cdot 1 + 0 + \frac{M_0}{P} \cdot 3$$

$$\boxed{C_1 = -\frac{M_0}{P}}$$

Differentiating Eqn (3) with respect to x .

$$\frac{dy}{dx} = C_1 (-1) \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right)$$

$$\sqrt{\frac{P}{EI}} + 0$$

$$= C_1 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) \sqrt{\frac{P}{EI}} \quad (4)$$

To sub $x = 0; \frac{dy}{dx} = 0$

$$0 = 0 + C_2 \cos(1) \sqrt{\frac{P}{EI}}$$

$$C_2 \sqrt{\frac{P}{EI}} = 0$$

$$\sqrt{\frac{P}{EI}} = 0$$

To sub the $C_1 = -\frac{M_0}{P}; C_2 = 0$ in eqn (3)

$$y = -\frac{M_0}{P} \cos\left[x \cdot \sqrt{\frac{P}{EI}}\right] + 0 + \frac{M_0}{P}$$

$$= -\frac{M_0}{P} \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \quad (5)$$

To sub the $x = l, y = 0$ in eqn (5)

$$0 = -\frac{M_0}{P} \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

$$+ \frac{M_0}{P} = \frac{M_0}{P} \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right)$$

$$\frac{M_0}{P} \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \frac{M_0}{P}$$

$$\cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \frac{M_0}{P} \times \frac{P}{M_0} = 1$$

$$l \cdot \sqrt{\frac{P}{EI}} = \cos^{-1}(1) \cos 2\pi \cdot (\cos 4\pi \cdot \cos 6\pi)$$

$$l \cdot \sqrt{\frac{P}{EI}} = 0, 2\pi, 4\pi, 6\pi \dots$$

Taking the least value ⁽⁷⁾

$$l \cdot \sqrt{\frac{P}{EI}} = 2\pi$$

$$\sqrt{\frac{P}{EI}} = \frac{2\pi}{l}$$

$$\frac{P}{EI} = \frac{4\pi^2}{l^2}$$

$$\therefore P = \frac{4\pi^2 EI}{l^2}$$

Expression for crippling load when one end of column is fixed and other end of column is hinged:-

Consider a column AB of length 'l' and uniform cross area fixed at the end A and hinged at the B as shown in fig.

Let P = crippling load, M_0 is the fixed end moment at the fixed end A and also horizontal reaction 'H' at top end B.

The moment at the section = moment due to crippling load at B + moment due to horizontal reaction at B.

$$M = -P \cdot y + H(l-x) \quad \text{--- (1)}$$

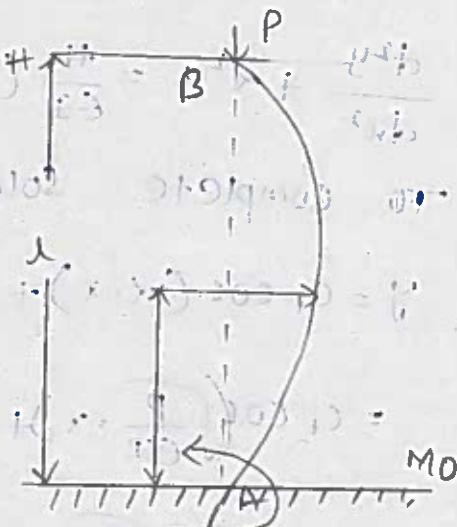
To consider the double integration

$$M = EI \frac{d^2y}{dx^2} \quad \text{--- (2)}$$

$$EI \frac{d^2y}{dx^2} = -P \cdot y + H(l-x)$$

$$EI \frac{d^2y}{dx^2} + P \cdot y = H(l-x)$$

Divide the L.H.S. on both sides



$$\frac{EI}{\text{constant}} \frac{d^2y}{dx^2} + \frac{P \cdot y}{EI} = \frac{H}{EI} (l-x)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{H}{EI} (l-x) \\ = \frac{H}{EI} (l-x) \frac{P}{P}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{P}{EI} \frac{H(l-x)}{P}$$

$$EI \frac{d^2y}{dx^2} = -P \cdot y + H(l-x)$$

$$EI \frac{d^2y}{dx^2} + P \cdot y = \frac{H}{P} (l-x)$$

Equation is written as

$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{H}{EI} (l-x) \quad \left[\alpha^2 = \frac{P}{EI}, \alpha = \sqrt{\frac{P}{EI}} \right]$$

To complete solution equation

$$y = c_1 \cos(\alpha \cdot x) + c_2 \sin(\alpha \cdot x) + \frac{H(l-x)}{EI \times \alpha^2} \\ = c_1 \cos\left(\sqrt{\frac{P}{EI}} \cdot x\right) + c_2 \sin\left(\sqrt{\frac{P}{EI}} x\right) + \frac{H(l-x)}{EI \times P / EI} \\ = c_1 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) + c_2 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) + \frac{H}{P} (l-x) \quad \textcircled{3}$$

Sub $x=0, y=0$ in eqn \textcircled{3}

$$0 = c_1(1) + 0 + \frac{H}{P} (l-0)$$

$$\boxed{c_1 = -\frac{H}{P} (l)}$$

To differentiate the eqn \textcircled{3} with respect to x .

$$\frac{dy}{dx} = -c_1 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) \sqrt{\frac{P}{EI}} + c_2 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \\ \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

At $x=0, \frac{dy}{dx} = 0$ in above the equation

$$0 = 0 + c_2(1) \int \frac{P}{EI} - \frac{\#}{P}$$

(18)

$$\frac{\#}{P} = c_2 \sqrt{\frac{P}{EI}}$$

$$c_2 = \frac{\#}{P} \sqrt{\frac{EI}{P}}$$

$$\boxed{\frac{\#}{P} \sqrt{\frac{EI}{P}} = q}$$

To sub the eqn $\# = \frac{\#}{P} x$, $c_2 = \frac{\#}{P} \int \frac{EI}{P}$

equ (3), we get

$$y = -\frac{\#}{P} l \cos\left(x \sqrt{\frac{P}{EI}}\right) + \frac{\#}{P} \int \frac{EI}{P} \sin\left(x \sqrt{\frac{P}{EI}}\right) + \frac{\#}{P} (l-x) \quad \text{--- (4)}$$

To sub the $x=l$, $y=0$ in equ (4)

$$0 = -\frac{\#}{P} l \cos\left(l \sqrt{\frac{P}{EI}}\right) + \frac{\#}{P} \int \frac{EI}{P} \sin\left(l \sqrt{\frac{P}{EI}}\right)$$

$$\frac{\#}{P} \int \frac{EI}{P} \sin\left(l \sqrt{\frac{P}{EI}}\right) = \frac{\#}{P} l \cos\left(l \sqrt{\frac{P}{EI}}\right)$$

$$\sin\left(l \sqrt{\frac{P}{EI}}\right) = \frac{\#}{P} \cdot l \cdot \frac{P}{H} \times \sqrt{\frac{P}{EI}} \cos\left(l \sqrt{\frac{P}{EI}}\right)$$

$$\frac{\sin\left(l \sqrt{\frac{P}{EI}}\right)}{\cos\left(l \sqrt{\frac{P}{EI}}\right)} = l \sqrt{\frac{P}{EI}}$$

$$\tan\left(l \sqrt{\frac{P}{EI}}\right) = l \sqrt{\frac{P}{EI}}$$

To consider the $\tan\left(l \sqrt{\frac{P}{EI}}\right)$ is small qty

$$r = l \cdot \sqrt{\frac{P}{EI}}$$

$$\text{The above the solution} = l \cdot \sqrt{\frac{P}{EI}} = 4.5$$

$$l \cdot \frac{P}{EI} = 4.5 \text{ radians}$$

squaring on b.s

$$l^2 \left(\sqrt{\frac{P}{EI}}\right)^2 = 4.5^2$$

$$P = 20.25 \frac{\epsilon I}{l^2}$$

But approximately $20.25 = 2\pi^2$

$$\therefore P = \frac{2\pi^2 EI}{l^2}$$

Assumptions made in Euler's theory:

- A column initially perfectly straight and load is applied axially.
- the cross section of the column is uniform throughout its length.
- The self weight of the column is negligible.
- The column will fail by buckling alone.
- The length of column very large as compared to its lateral dimensions.
- The direct stress is very small as compared to the bending stresses.
- The column material is perfectly elastic, homogeneous and Isotropic and obeys hooks law.

(19) 20.01.08

Limitations of Euler's theory:

- Euler's formula is used for long columns and neglects the stress due to direct compressive loads.
- If the slenderness ratio is less than 80, the Euler's formula for mild steel column is not valid.
- In Euler's formula, that the critical (or) allowable stress on a column decreased with the increase due to the slenderness ratio.
- Long axial long columns tends to deflect about the axis of least moment of inertia, the least radius of gyration and it should be used for determine the slenderness ratio.

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

≈ 3000

≈ 3000

problems:-

- 1) A solid round Bar 3m long and 5cm dia. is used as a strut with both ends are hinged. Determine the crippling (or) collapsing load. Take $E = 2 \times 10^5 \text{ N/mm}^2$.

Given data:

$$\text{length } (l) = 3\text{m}$$

$$\text{dia } (d) = 5\text{cm} = 50\text{mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

both ends are hinged.

$$L = l = 3000 \text{ mm}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$I = \frac{\pi}{64} \times D^4$$

$$= \frac{\pi}{64} \times 50^4$$

$$= 30676.157 \text{ mm}^4$$

$$= 30.68 \times 10^4 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2}$$

$$= 67288.76 \text{ N}$$

- (2)
- 2) A column of timber section: 15cm x 20cm is 6 m long both ends fixed. The young's modulus of timber = 17.5 kn/mm². Determine the 1) crippling load
2) safe load for the column of factor of safety = 3.

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Given data:

Timber section: 15cm x 20cm

$$l = 6 \text{ m} = 180 \text{ mm} \times 200 \text{ mm}$$

$$E = 17.5 \text{ kn/mm}^2 = 17.5 \times 10^3 \text{ N/mm}^2$$

$$F.O.S = 3$$

$$Kl = l = 1/2 = \frac{6000}{2} = 3000 \text{ mm}$$

$$I = \frac{bd^3}{12} = \frac{150 \times 200^3}{12} = 10000 \times 10^4 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2 \times 17.5 \times 10^3 \times 10000 \times 10^4}{3000^2}$$

$$= 1919089.745 \text{ kn.}$$

Safe load = critical load

F.O.S

$$= \frac{1919089.745}{3}$$

$$\text{Safe load} = 639696.5817 \text{ kn}$$

From Poix CSU-31

nothing to timber load

Yot x H.B

cc 21 ✓

Rankine's formula:-

$$P = \sigma_c \cdot A$$

$$\frac{1}{1+a} \left[\frac{\sigma_c}{E} \right]^2$$

where, σ_c = Rankine's constant.

$$a = \frac{\sigma_c}{\pi^2 E}$$

problems:-

- 1) The external and internal dia of a hollow cast iron column are 5cm and 4cm respectively. If the length of the column is 3m and both of its ends fixed. Determine the crippling load using Rankine's formula. Take this values of $\sigma_c = 550 \text{ N/mm}^2$ and $a = \frac{1}{1600}$ in Rankine's formula.

$$D = 5 \text{ cm} = 50 \text{ mm}$$

$$d = 4 \text{ cm} = 40 \text{ mm}$$

$$l = 3 \text{ m} = 3000 \text{ mm}$$

$$\sigma_c = 550 \text{ N/mm}^2$$

$$a = \frac{1}{1600}$$

$$A = \frac{\pi}{4} (D-d)^2 = \frac{\pi}{4} (50-40)^2 = 78.53 \text{ mm}^2$$

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (50^4 - 40^4)$$

$$I = 181132.45 \text{ mm}^4$$

$$= 18.1132 \times 10^4 \text{ mm}^4$$

∴ least radius of gyration

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{18.11 \times 10^4}{78.53}}$$

$$k = 48.022$$

(21)

Effective length $kL = l/2 = \frac{3000}{2} = 1500\text{ mm}$

$$\begin{aligned}
 P &= \sigma_c \cdot A \cdot \left[1 + \frac{1}{1+a} \left(\frac{Le}{E} \right)^2 \right] \\
 &= \frac{550 \times 78.53}{1 + \frac{1}{1600} \left(\frac{1500}{48.022} \right)^2} \\
 &= 26.830 \times 10^3 \text{ kN}
 \end{aligned}$$

Straight line formula:

The empirical straight line formula are commonly used in practically in design.

$$P = \sigma_c \times A - n \left[\frac{Le}{E} \right] \times A$$

Johnson's parabolic formula:

$$P = \sigma_c \cdot A - r \left[\frac{Le}{E} \right] \times A$$

The crippling load according to the Johnson, is given by equation.

Indian standard code for mild steel by using formula [secant formula]

$$\begin{aligned}
 \sigma_c = \sigma_c' &= \frac{\sigma_y}{n + 0.20 \sec \left[\frac{Le}{E} \sqrt{\frac{m\sigma_c'}{4E}} \right]} \\
 &\quad [\because \frac{Le}{E} = 0 \text{ to } 160]
 \end{aligned}$$

$$\sigma_c = \sigma_c' \left[1.2 - \frac{Le}{800k} \right]$$

$$[\because \frac{Le}{E} = 160 \text{ & above}]$$

where $\sigma_y = \text{min yield stress taken as } 250 \text{ N/mm}^2$

m = factor of safety as taken 1.68
 ℓ/k = slenderness ratio

E = modulus of elasticity / young's modulus
($2 \times 10^5 \text{ N/mm}^2$)

σ_c' = A value obtained from the above secant formula.

σ_c = allowable axial compression stress obtained from slenderness ratio.

Beam - columns:-

columns carry axial compressive loads.

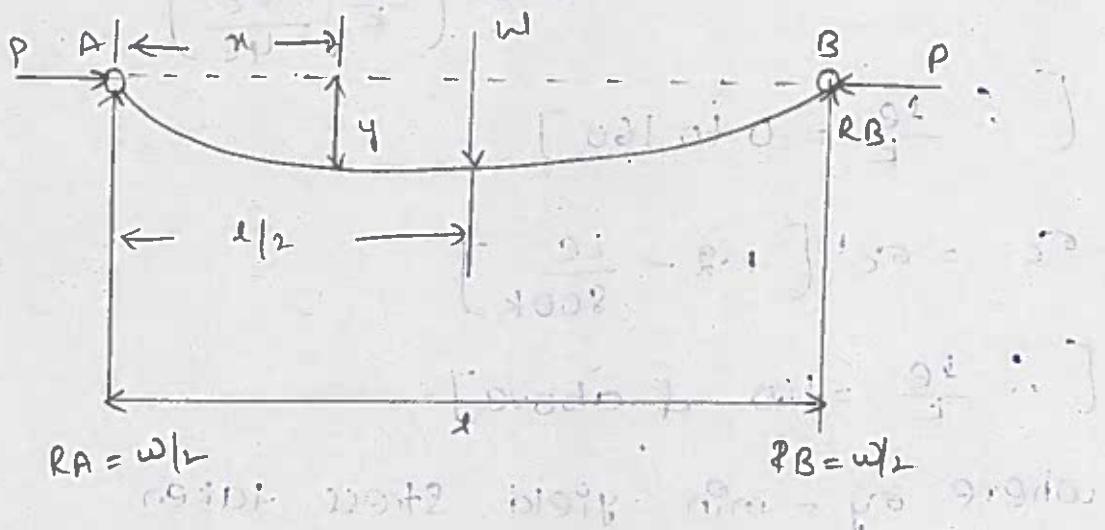
If the columns are also subjected to traverse load then they are known as Beam columns. A traverse load is generally uniformly distributed.

But let us consider 2 cases.

Case(1) :- Traverse load is a point load and act at the center.

Case(2) :- traverse load is uniformly distributed.

Case(1) :- Traverse load is a point load and act at the center.



(22)

By using double integration

$$M = E \cdot I \frac{d^2y}{dx^2} \quad \text{--- (1)}$$

The B.M at the section is given by,

$$M = -P \cdot y = \frac{\omega}{2} \cdot x \quad \text{--- (2)}$$

To evaluate the eqn (1) & (2)

$$E \cdot I \frac{d^2y}{dx^2} = -P \cdot y - \frac{\omega}{2} \cdot x^2$$

$$\frac{d^2y}{dx^2} = \frac{-P}{EI} \cdot y - \frac{\omega}{2EI} \cdot x^2$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = -\frac{\omega}{2EI} x^2$$

The solution of the above differential equation is

$$y = c_1 \cos \left(x \times \sqrt{\frac{P}{EI}} \right) + c_2 \sin \left(x \times \sqrt{\frac{P}{EI}} - \frac{\omega x}{2EI} \right)$$

$$y = c_1 \cos \left[x \times \sqrt{\frac{P}{EI}} \right] + c_2 \sin \left[x \times \sqrt{\frac{P}{EI}} \right] - \frac{\omega x^2}{2P} \quad \text{--- (3)}$$

To differentiating the above eqn (3)
with respect to x .

$$\frac{dy}{dx} = -c_1 \sqrt{\frac{P}{EI}} \sin \left(x \times \sqrt{\frac{P}{EI}} \right) + c_2 \sqrt{\frac{P}{EI}}$$

$$\cos \left(x \times \sqrt{\frac{P}{EI}} - \frac{\omega x}{2P} \right) \quad \text{--- (4)}$$

The values of c_1 and c_2 are obtained from boundary conditions.

at $x=0$, $y=0$ hence from eqn (3) we will get

$$0 = c_1 (1) + 0 \Rightarrow 0 = 0$$

$$\boxed{c_1 = 0}$$

At $x = l/2$, $\frac{dy}{dx} = 0$, $c_1 = 0$. Hence from

eqn (4), we will get

$$0 = 0 + C_2 \sqrt{\frac{P}{EI}} \cos \left(\omega t_2 \sqrt{\frac{P}{EI}} \right) - \frac{\omega}{2P}$$

$$C_2 = \frac{\omega}{2P} \sqrt{\frac{EI}{P}} \times \frac{1}{\cos(\omega t_2 \sqrt{\frac{P}{EI}})}$$

$$C_2 = \frac{\omega}{2P} \sqrt{\frac{EI}{P}} \sec \left(\omega t_2 \sqrt{\frac{P}{EI}} \right)$$

Sub the value of C_1 and C_2 in eqn ③ or we get

$$y = 0 + \frac{\omega}{2P} \times \sqrt{\frac{EI}{P}} \sec \left(\omega t_2 \sqrt{\frac{P}{EI}} \right)$$

$$\sin \left(x \times \sqrt{\frac{P}{EI}} \right) = \frac{\omega x}{2P}$$

$$y = \frac{\omega}{2P} \sqrt{\frac{EI}{P}} \sec \left(\omega t_2 \sqrt{\frac{P}{EI}} \right) \sin \left(x \times \sqrt{\frac{P}{EI}} \right) - \frac{\omega x}{2P}$$

To sub $x = l/2$ in eqn ③

$$y = \frac{\omega}{2P} \sqrt{\frac{EI}{P}} \sec \left(\omega t_2 \sqrt{\frac{P}{EI}} \right) \sin \left(\frac{l}{2} \sqrt{\frac{P}{EI}} \right) - \frac{\omega(l/2)}{2P}$$

$$y_{max} = \frac{\omega}{2P} \times \sqrt{\frac{EI}{P}} \tan \left(\omega t_2 \sqrt{\frac{P}{EI}} \right) - \frac{\omega l}{4P} \quad \text{④}$$

Max: Bending moment

The B.M is given by eqn ④ as

$$\begin{aligned} M &= -P \cdot y - \frac{\omega}{2} \cdot x \\ &= -(P \cdot y + \frac{\omega}{2} \cdot x) \end{aligned}$$

The B.M will be max at the centre

where $y = y_{max}$ and $x = l/2$ sub the above the eqn.

$$= -(P \cdot y_{max} + \frac{\omega}{2} \cdot l/2)$$

$$= \left[P \times \left(\frac{\omega}{2P} \times \sqrt{\frac{EI}{P}} \times \tan \left(\omega t_2 \sqrt{\frac{P}{EI}} \right) - \frac{\omega l}{4P} \right) + \frac{\omega l}{4} \right]$$

$$= - \left[P \frac{\omega}{2P} \times \sqrt{\frac{EI}{P}} \times \tan \left(\omega t_2 \sqrt{\frac{P}{EI}} \right) - \frac{\omega l}{4} + \frac{\omega l}{4} \right]$$

$$M = - \left[\frac{\omega}{2} \sqrt{\frac{EI}{P}} \times \tan \left(\frac{1}{2} \sqrt{\frac{P}{EI}} \right) \right]^{23}$$

The (-ve) sign is due to sign conversion.
Hence the magnitude of max. bending moment
is given by

$$M_{\max} (\text{magnitude}) = \left[\frac{\omega}{2} \cdot \sqrt{\frac{EI}{P}} \times \tan \left(\frac{1}{2} \sqrt{\frac{P}{EI}} \right) \right] \text{---} \oplus$$

Max. stress (σ_{\max})

Max. stress induced is due to direct axial compressive load and due to max. bending stress.

$$\sigma_{\max} = \sigma_o + \sigma_b$$

$$= \frac{P}{A} + \sigma_b.$$

The stress due to bending of strut is

$$\text{given by } \frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M}{I} \times y = \frac{M \times y_c}{Ak^2}$$

where, y_c = distance of the extreme layer in compression from neutral axis.

$$I = Ak^2$$

$$k = r_{min}$$

$$M = M_{\max}$$

Max. Bending stress

$$\begin{aligned} \sigma_b &= \frac{M_{\max} \times y_c}{Ak^2} \\ &= \frac{\left[\frac{\omega}{2} \sqrt{\frac{EI}{P}} \tan \left(\frac{1}{2} \sqrt{\frac{P}{EI}} \right) \right] \times y_c}{Ak^2} \end{aligned}$$

Hence max. stress induced becomes

$$\sigma_{\max} = \frac{P}{A} + \left[\frac{\omega}{2} \times \sqrt{\frac{EI}{P}} \times \tan \left(\frac{1}{2} \times \sqrt{\frac{P}{EI}} \right) \right] \times \frac{y_c}{A^2}$$

(Ans)

(Ans) 120.9 N/mm

Max. stress at mid span is 120.9 N/mm
calculated from the free body diagram

(Ans)

(Ans)

$$= 120 + \frac{9}{4}$$

Max. stress at mid span is 120.9 N/mm

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

$$M \times M = M^2$$

Max. stress at mid span is 120.9 N/mm
calculated from the free body diagram

(Ans)

(Ans)

(Ans)

Max. stress at mid span is 120.9 N/mm

Max. stress at mid span is 120.9 N/mm

(Ans)

$$120 \times \left(\frac{9}{4} + \frac{1}{4} \right) \cos \left(\frac{1}{2} \times \frac{\omega}{\sqrt{EI}} \right)$$

(Ans)